**Ross Program Application Essays**

**1. Problems**

**What is an interesting mathematical problem you have worked on? Recall a problem that you spent some time thinking about. Carefully state the problem. Describe the work you have done on it.**

“One, two, three, …” I was trying to draw out the complete exchange graph of Mobius stripe with three marked points, wondering whether it was possible to find the number of all possible triangulations. Easy as I initially thought it would be, it turned out to be much less intuitive. Frustrated, I turned to Cluster Algebra in the hope of viewing this intriguing problem from a different perspective: I drew their exchange graphs and cluster complexes, and represented them in double cover, snake graph...

I also tried using brute force up to 4 marked points, but the result turned out to be so messy and overwhelming. When I was nearly on the verge of emotional collapse, my mentor came up with an idea, “Maybe we can look at this from a combinatorics perspective, and it might have something to do with Catalan number.” I was almost suddenly enlightened: in each triangulation, each boundary edge is part of only one triangle. Inspired by this idea, we soon sorted out the three cases in which two arcs could divide the non- orientable surface. For the orientable part, which is the area without the crosscap, we directly apply Catalan number to count the number of triangulations. For the non-orientable part, which is the area with the crosscap, we defined the number of triangulations recursively. By adding the three cases, we got the final formula for the answer of the question.

However, the answer we got was recursively defined, and a more simple function seemed difficult to find – even wolfram alpha cannot interpret a recursive formula with that complexity. I have proposed a new, simple formula based on observation, but its proof – either through the simplification of the recursive definition or by thinking from another perspective – awaits me to explore.

Logic and rigor have always been the inherent charm of mathematics, but what is more attractive to me is the unexpectedness in mathematics. It seems to be waiting for me right around the corner, guiding me to link everything together but think out of the box at the same time.

**2. Projects**

**Have you worked on some interesting mathematical projects? If so, what were the topics? For the project you enjoyed the most: How much time did you spend on it? Were you working alone, guided by a mentor, or as part of a team? What benefits do you feel you gained from doing that work?**

Collaborative: The unistructurality of quasi-cluster algebra; The number of triangulations on a Mobius stripe;

Individual: Tensor calculus and Kaluza theory; Mathematical model of local climate

The one I enjoyed most was the proof of the unistructurality of quasi-cluster algebras.

We were two students mentored by a PhD. I first read about the unistructurality of cluster algebras, and then I began to wonder what will happen if a cluster algebra is on a non-orientable surface like a Mobius stripe – will the unistructurality still hold?

I first turned this into a problem about triangulation: in order to prove the unistructurality, all I need to do is to prove that a cluster algebra does not depend on the particular triangulation we start from. Then I could get two important formulas from the positivity, which will help me prove the unistructurality of quasi-cluster algebras.

However, when I shared my ideas to my colleagues, my mentor told me that my proof had a flaw - I haven’t proved that positivity holds for quasi-cluster algebra yet. However, my proof, starting from lemma, was so based on the validity of positivity. We were pretty sure that positivity should be true for quasi-cluster algebra also, but none of us had any clue of how to prove it.

I read all existing literature I could find about quasi-cluster algebras, but it is a relatively new area so there are not many resources available. After two months of working on this question, my mentor excitedly showed us a new article published several days ago, proving the positivity for quasi-cluster algebras. The author uses a very clever method that neither of us even thought about: snake graphs and lamentation.

I was first amazed that someone was thinking about the same question as I did, so I contacted him after reading his paper. He was happy to hear from me and was willing to help me with my further investigation on basis, which is a new area in Cluster Algebra that I am growing interest in. Meanwhile, through this project, I got a deeper understanding of how math research is like. I could see the how the huge mathematical knowledge system is constantly built at such an explosive speed, with all the intellectuals from the world specializing in different math topics. As the most direct outward demonstration of human intelligence and our consistent pursuit for perfection, math never fails to flood me with awe.

**3. Other programs**

**Have you participated in academic programs outside of school? This might include another summer math camp, a local Math Club, or a Math Circle at some nearby college. What sorts of math activities were involved? Did you enjoy those experiences?**

“Mathematics all day, every day.”

This was my morale chant at Canada/USA Mathcamp last summer. I spent all day swimming in the ocean of mathematics and exposed myself to college-level mathematics for four consecutive weeks, including but not limited to Cluster Algebra, Knot Theory, Non-Euclidean Geometry, Root System, Morse Theory. Often, my roommate fell into sleep with her Poincare Formula worksheet; I woke up where she left off. The first second we looked at each other, we would begin to discuss our new ideas. I walked with mathematics, showered with mathematics, and ultimately lived with mathematics. What had long been treated oddly in my previous life had finally become even: my peers joined in instead of watching bewildered while I constructed all sorts of triangles with my hands at meals.

“How to prove 2+2=4?” I was lying in my dorm back in Victoria, gazing at the floor covered by scratch paper, with this single straightforward yet difficult problem stuck in my mind in the last two days. I always carry the spirit of my Mathcamp experience with me. In this hi-tech age, the simple trio of paper, pen, and determination is still capable of surprising insight. And along the way, it feels good to be confused and satisfying to tackle the confusion.

“All day, every day.”

**4. Competitions**

**Have you recently participated in some math competitions? Which ones? Did you do well on them? How do your math contest experiences (both preparing for and participating in contests), compare with math courses you have taken in high school or in other venues?**

Last month I got invited to visit the University of British Colombia because of my excellent performance in the Canadian Open Mathematics Challenge. It turned out that I got an Honor Award at the 11th-grade level. Recently, I also received a “Meritorious” award for HiMCM that I paid efforts for with other three friends last November.

In January, I participated in the MIT puzzle hunt online, which was the most fun competition I’ve ever attended. I’d rather call it less a competition than a gathering where a group of math lovers solved interesting problems together. It didn’t matter whether who win or lose. In fact, the math shouldn’t be competitions which are used to evaluate computing ability with limited time.

Math is more about creativity and faith. When I prepare for proving a mathematical formula, I need to know the logic behind the proofs first and conceive what mathematical tools I’m going to use and how I’ll use them. In most cases, it does require an answer or a statement; however, what counts more is the logic behind and how I get it. Only when I slow down, deliberate every detail in a problem can I fully appreciate the beauty of mathematics.

Other Math Honors and Awards:

1) Meritorious Award, 22nd Annual High School Mathematical Contest in Modeling (HiMCM), 02/2020;

2) Honor Roll in BC, the Canadian Open Mathematics Challenge, qualified for attending Canadian Mathematical Olympiad (CMO), 11/2019;

3) Top 5%, Euclid Contest, The CENTER for EDUCATION in MATHEMATICS and COMPUTING, University of Waterloo, 05/2019;

4) First Prize (China), Math League, qualified for US Final (at Stanford) and its Math Camp (co-held by the Departments of Mathematics of Princeton University, Columbia University, and Williams College), 2018;

5) Second Prize, Jiangsu Junior High School Mathematics Competition, 2017;

6) Second Place (Global Final), The Berkeley Mini Math Tournament (BmMT), 2017.

**5. Books and websites**

**What have you read recently about mathematics? Which math books are your favorites? What parts of them were most enjoyable or interesting to you? What interesting mathematical websites have you visited in the past year? What aspects of math have you learned by reading the material on those sites?**

**Books:**

1) What Is Mathematics - *Richard Courant; Herbert Robbins*

2) Geometry of Surfaces - *John Stillwell*

3) Studies on Paradoxes – *Bo Chen*

4) The Knot Book- *Colin Adams*

5) Mathematics and Philosophy – *Jingzhong Zhang* (The book enumerates how mathematical tools are used to solve questions of philosophical significance, and discusses different philosophical perspectives of mathematics. Numerous examples in the book inspire me to perceive math from a brand new angle - what it is, how it was born and developed.)

6) Mathematics + Art: A Cultural History – *Lynn Gamwell*

7) Gödel, Escher, Bach: An Eternal Golden Braid - *Douglas Hofstadter* (Gödel’s incompleteness theorems, Escher’s paradoxical painting style, and Bach’s fugue musical compositions inspire me to think beyond dualism rather than endlessly examine the complete opposition between truth and paradox, formalism and informalism, the human brain and artificial intelligence.)

**Websites:**

1) Wolfram MathWorld (Fabulous math encyclopedia where I’ve learned mathematical equations and theories I encounter and have access to an abundant resource for research.)

2) Brilliant (The online community empowers me to build quantitative skills in math, science, and computer science with fun and challenging interactive explorations.)

3) MIT OpenCourseWare (An inspiring source for me to explore mathematics at a college-level, for example, root system in a Euclidean space, Young Tableaux in the course of combinatorics, and introduction into the representation theory.)

4) Expii (Learn math creatively.)

5) Global Math Project

6) Association for Women in Mathematics (AWM)

7) Art of Problem Solving (AoPS is a cornucopia where I challenge myself with various tough mathematical problems.)

**6. Future goals**

**What do you plan to major in at college? What are your career goals? Do you feel “driven” toward one type of work? (Like an academic career in math, astronomy, or economics? Or a career in finance? etc.) Or will you take a variety of courses in college and see what areas seem most interesting?**

“The water waves shone in aqua, the light pours down through the traditional-style railings…” It was a model we built with recycled materials for a complex on our campus. The waves were tailored from discarded Sprite bottles, and the buildings were made of used cardboard.

The complex was designed by a disciple of Ieoh Ming Pei. Inspiration coming from classical Chinese gardens, it not only reserves the slanting roofs and enclosed courtyards but also what I call “mooring in space.”

During my childhood, I liked to wander around Suzhou’s historic gardens. There’s one with a pond and a bridge at one end. While viewed from a certain angle, the pond extends beyond the bridge and into the bamboo forest. Everything then builds upon one another visually. When I look through a floral window or walk along an ambulatory, it gives a sense of infinity for the path winding into jungles.

I’d liken these multi-dimensional garden architectures to the polytopes I learned in mathematics - objects travel in four-dimension. Each part of the garden seems to be distinct and separate, but when viewed from the right angle, they become such a vibrant entirety. The parts travel, tangle, and scatter around. They interact in such a way that even could be interpreted by most advanced mathematical theory. But this architectural art goes far beyond the logic, turning mathematics from the rational to the perceptual, from the regular to the non-linear.

I plan to study mathematics and architecture at college, aspiring to be a designer who will mathematically interpret the architectural idea.

**Collaboration**

**When you work on hard math problems do you usually work alone? Or with a group of friends? Reminder: when you work on the Ross Application Problems, please be sure to work by yourself.**

I don’t mind working alone, and in fact, I usually work on math problems alone. Sometimes I enjoy this kind of loneliness since I could not find a peer as passionate about math as I am in my school. I’ve been accustomed to spending hours thinking about a problem myself before turning to my math teacher or senior students for help. This struggling process of figuring out a tough math problem can be likened to that of being suffocated under the water. Only after I learned to swim and can float on the surface of the water can I learn to appreciate the beauty of mathematics. Struggle and progress are twin brothers.

In some cases, however, that is not the entire truth. When investigating quasi-cluster algebra, I’d prefer to seek collaborations with other math friends. During the process of proving the number of triangulations, I discussed a lot with my mentor and like-minded peer via video meetings. Exchange with other math lovers, especially those who are adept in various fields of mathematics, could be extremely rewarding because *the fire burns high when everybody adds wood to it*. Collaboration can offer new insights and a broader horizon into a difficult yet intriguing problem. It is one of the reasons why I am so looking forward to attending the Ross Program, which will not only open for me the window of number theory but also provide a community to meet a group of young scholars sharing the same enthusiasm about mathematics.

**Other coursework**

**Have you taken math courses outside of the standard high school curriculum? Those courses could have been at a local college, in a residential math program, from an educational website, etc.**

Most of my acquisition of mathematical knowledge comes from outside of my school curriculum. I still remember the first time I saw a video from 3Blue1Brown, where I run into Linear Algebra and sensed the study of mathematics can not only be realized step by step following my school curriculum. After searching for mathematics courses from Coursera, edX, and Socratica Studios, I decided to self-study high school mathematics.

Mathcamp has once again changed my perception of math learning. The vast majority of advanced mathematical theories can be acquired in a basic way. Math is diverse; different topics and theories in fields can be interconnected. I began to understand the properties of linear transformation I met within the 3Blue1Brown videos. I was no longer amazed by the Lagrange theorem for its connecting number theory with group theory. I engaged myself to study how linear algebra can be used in the area of algebraic topology to prove the famous Euler-Poincare formula.

Enchanted by this charming formula in modern mathematics, I enrolled in an online course of algebraic geometry instructed by Visiting Professor Sheshmani at Harvard University. However, studying with a group of undergraduates, I realized I still have a lot to learn before jumping into advanced fields. Before entering an intense war, I need to do more strenuous physical training. Therefore, at the Ross Program, I look forward to building up further on my math knowledge system and exercising my “muscles” through a sea of problem sets.

**Being away from home**

**Are you eager to spend five or six weeks away from home, with no distractions from computers or video games or smart phones, focusing all of your energies on one narrow area of**

**mathematics? Ross students live in a college dormitory, with no access to televisions, computers, or electronic games. Most of their waking hours are spent working on challenging math problems. Does this intensity and focus appeal to you?**

I woke up with the howling of sea animals. It took a while before I realized being in an ecological reserve. It was still dark outside. I opened the door. The peculiar smell of coastal creatures hit me. Then, I walked out. Living on Race Rocks Islands for a week, I’d been used to a lot of things: no shower, frozen food, limited electricity… I only looked forward to the sunrise and sunset every day. “The demands of mankind are unlimited,” I started to reflect on what my economics teacher said in class. I’d like to get lost on the island, where I have little but lack nothing. Self-sufficiency is all I need, just like how it was since the dawn of mankind.

This time, I want to carry the same pureness to the Ross Program.

I’m excited about immersing myself in mathematics. Working on a math problem for an hour or two at night has become a daily routine for me. There are times when I’m so fixated on a problem that I’ll think about it for most of the evening, even when I’m brushing my teeth or lying on my bed before falling asleep. However, with patience and accumulation of information, I’ll piece together the clues to find a solution. I am expecting to have one period of time when I can totally focus myself on mathematics to gain momentum, to enjoy the process, and to fully research and delve into this topic.

Therefore, the Ross Program is such a wonderful opportunity for me to enjoy the purity of mathematics. I will study with math lovers from all over the world, exchange thoughts with counselors and professors of strong math backgrounds, and work on piles of problem sets for the whole day without worrying about any external distraction.